

Caution and Monopolistic Competition

Jaime Velasco Sánchez

University of Surrey

j.velascosanchez@surrey.ac.uk

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Abstract

When consumers are not sure how to compare brands, caution creates monopolistic competition. This is addressed by looking at an exchange economy with countably many differentiated goods and analysing the price impact of a coalition of countably many agents withholding their endowments. Such impact persists even when making the coalition smaller and smaller.

Motivation

- 1 How to draw boundaries between perfect and imperfect competition?
Is it **possible for an arbitrarily small coalition of agents to impact prices by withholding part of their endowments?**,
- 2 Being **unsure** towards consuming different brands of the same goods could cause a monopolistic competition outcome,
- 3 Perfect competition is usually portrayed using a continuum of agents (Aumann, (1964)). **Only with a countable infinite set of agents and brands we can capture either a monopolistic competition outcome or a perfect competition outcome**, depending on the withholding test result.

Outline

- 1 Caution and Reluctance to Enlarge the Predilected Set
- 2 Testing the Price Impact of Withholding Endowments
- 3 Results
- 4 Conclusion

Caution and Reluctance to Enlarge the Predilected Set

Ambiguity Aversion and Brands Assessment

We depart from standard preferences by assuming that **agents are not sure about how to compare brands**. This can be seen by taking as baseline utility an additively separable utility, and considering a set C of possible different sequences of weights.

- When not sure on how to weight the brands, the consumer takes a pessimistic approach and picks for each consumption bundle the weights process that gives her the lowest aggregate utility, within some set of possible weights processes (with a lower bound process) and then maximizes such minimal utility.

The **aversion to the ambiguity with respect to these weights** makes the consumer evaluate each consumption bundle $x \in \ell_{\infty}^+$ using the additively separable utility computed with the most penalising sequence of weights $\delta \in C$ for that bundle.

The above has two important implications:

- It is equivalent to maximizing the utilities weighted by such lower bounds plus another term which is the infimum of moving cumulative averages of unweighted utilities,
- The consumer is **cautious** about what might be the impact on her cumulative average utility of expanding consumption along the infinite set of brands.

Caution and Differentiation

Formally, preferences of agent i are represented by

$$U^i(x) = \sum_{j=1}^{\infty} \varsigma_j u(x_j) + \beta_i \inf_j \left(\frac{1}{j} \sum_{k=1}^j u(x_k) \right), \quad (1)$$

illustrating that the consumer is wary to add more brands to her list as she fears that it might decrease the average utility she has enjoyed so far along the list.

The last term in equation (1) has great similarities with the Polya Index introduced by Marinacci (1998). These preferences can be shown to exhibit aversion to the ambiguity on how to downgrade elements further down on the agents' list, suggesting habit persistence as a result.

We claim that equation 1 reflects **precautionary behaviour** on the preferences of our agents:

- ① First summand: weights ς_j for each $u(x_j)$,
- ② Second summand: we have another weight for merchandises $j \in \{1, \dots, m\}$. Such weight is $\frac{\beta^i}{m}$, making $\delta_j = \varsigma_j + \frac{\beta^i}{m} \ \forall j \in \{1, \dots, m\}$.

The second summand is where **caution** appears:

- The agent is unsure on how to weight the utility perceived from consuming brands further in the list indexed by $m+1, \dots$,
- Since she is cautious about consuming such goods, she is **wary about how consuming these merchandises affects her mean utility**, portrayed by taking the worse case scenario given by the infimum.

More on the Economic Interpretation of U^i

The Economy

We assume that each agent i is endowed with only a positive quantity ω_i^i of her agent-specific commodity. Let

$$\omega = (\omega_i^i)_{i \in \mathbb{N}}$$

be the endowment of the whole economy, with

$$\omega^i = (0, 0, \dots, 0, \underbrace{\omega_i^i}_{\text{i-th entry}}, 0, 0, \dots).$$

With countably many agents, the space of commodities is ℓ_∞^+ .

An economy is a triplet made of the commodity space (X), preferences ($\geq$) and endowments (ω), i.e. $\mathcal{E} = (X, \geq^i, \omega^i)_{i \in I}$. In our case,

$$\mathcal{E} = (\ell_{\infty}^+, U^i, \omega^i)_{i \in \mathbb{N}}. \quad (2)$$

- Equilibrium prices, π , and allocation, $(x^i)_{i \in \mathbb{N}}$, exist for $\mathcal{E}$,
- Bewley (1972) proved the existence of equilibrium, while Araujo et al. (2019) use it in a dynamic setting with habit persistence,
- Preferences in Araujo et al. (2019) are formally analogous to ours.

A Closer Look to Prices

The **price functional** π of the AD equilibrium belongs to $ba(2^{\mathbb{N}})$. By the Yosida-Hewitt Decomposition, we can represent such prices as $\pi = \mu + \nu$, where μ is a *countably additive part* and ν is the *purely finitely additive part*.

The **AD prices at x^1 when evaluated at direction y** are as follow

$$\pi(y) = \sum_{j=1}^{\infty} \varsigma_j u'(x_j^1) y_j + \beta_i u'(x^1) \text{LIM}(\phi(y))$$

where $\phi: \ell_{\infty}^+ \rightarrow \ell_{\infty}^+$ such that $\phi(y)_j = \frac{1}{j} \sum_{k=1}^j y_k$, and LIM represents a linear functional taking on each $y \in \ell_{\infty}$ a value in $[\liminf y, \limsup y]$ and x^1 is the AD equilibrium bundle for agent 1, as in Araujo, Gama, Novinski and Páscoa (2019).

Testing the Price Impact of Withholding Endowments

MC and Market Power

From “*Theory of Monopolistic Competition*” (Chamberlin, 1933), we can state that the characteristics of Monopolistic Competition structure are:

- Differentiated products,
- Large number of firms,
- Firms have market power.

Each of the firms have *some* monopolistic power. The power of monopoly resides on the fact that they can affect equilibrium prices of the market. Therefore, we define **market power** as the agents' ability to affect equilibrium prices.

How to test for market power in economy $\mathcal{E}$?

Our Withholding Test

My withholding test:

- 1 Let a coalition W_k of countably many agents withhold c^i , with $\inf_{i \in \mathbb{N}} \{\omega_i^i\} > c^i > 0$ (or $0 < c^i < \omega_i^i$), of their endowments,
 How to Construct the Withholding Coalition?
- 2 Calculate equilibrium prices for the new economy where a coalition of agents is withholding part of their endowments, i.e. equilibrium prices π_{W_k} in $\mathcal{E}_{W_k}$,
- 3 Shrink coalition W_k down to disappearing. Thus, generating a sequence of economies $(\mathcal{E}_{W_k})_{k \in \mathbb{N}}$, and calculate the limit of their equilibrium prices, i.e. $\lim_{k \rightarrow \infty} \pi_{W_k}$,
- 4 Is the price impact persistent even when the coalition shrunk down to disappearance?

Examples of Equilibria with Withholding

Inspiration

We take inspiration from Ostroy and Zame's (1994) pure exchange economy with a continuum of traders and commodities.

Their test: A coalition of traders withhold the endowments. Does the price impact disappear as the coalition shrinks to an individual agent?

Authors gave some assumptions on preferences and endowments under which one can have a persistent or a vanishing price impact.

Main differences between their withholding test and ours:

- 1 Countably many agents vs Continuum of agents,
- 2 Countably many goods vs Continuum of goods,

Our question

The question we want to answer with the withholding test is:

As agents withhold their endowments, do the increases in the prices of the agent-specific commodities tend to zero, as $k \rightarrow \infty$? We face two potential scenarios:

- If the price increase tends to zero, we have a **perfect competition** structure over the economy. Otherwise, there is **monopolistic competition**.

Results

Withholding Test

Proposition

Let agents $i \in W_k$ withhold from their endowments c^i , such that $0 < c^i < \inf_{j \in \mathbb{N}} \{\omega_j\} \ \forall i \in W_k$ (or $0 < c^i < \omega_j^i \ \forall i \in W_k$). Assume that for every agent i , the infimum $\underline{x}^i$ is not attained for every perturbed economy $\mathcal{E}_{W_k}$, with $k \in \mathbb{N}$.

- *When the withholding coalition is shrunk down to vanishing, (i.e. $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ with $\lim_{k \rightarrow \infty} W_k = \emptyset$), the equilibrium prices in $\mathcal{E}_{W_k}$ are higher than those in $\mathcal{E}$ for every $k \in \mathbb{N}$.*

Idea of the proof:

- 1 For a $y \in \ell_\infty^+$, take $\lim_{k \rightarrow \infty} \pi_{W_k}(y)$, where the impact of withholding is reflected on $u'(\underline{x})$, given its concavity.
- 2 Compare $\lim_{k \rightarrow \infty} \pi_{W_k}(y)$ in $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ with $\pi(y)$ in $\mathcal{E}$.

Conclusion

Conclusion

- **Caution**, as formulated in this work, is one of the reasons monopolistic competition can arise.
- We have two examples:
 - ① A tail, beyond some index k , withholding (finite agents not withholding),
 - ② Agents indexed by an even number withholding (countably many agents not withholding).
- Even when there are countably many agents *not withholding*, withholding coalitions can have a **persistent price impact**.

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Wariness and Capacities

Definition

A **capacity** is a function $\nu : 2^{\mathbb{N}} \rightarrow \mathbb{R}$ such that $\nu(\emptyset) = 0$ and $\nu(A) \leq \nu(B)$ whenever $A \subseteq B$.

For our case, we normalize $\nu(\mathbb{N}) = 1$.

Definition

The set **core**(ν) is defined as $\{\eta \in ba : \eta \geq \nu, \eta(\mathbb{N}) = 1\}$.

Definition

A capacity ν is **continuous at certainty** if, for any sequence $(A_n)_{n \in \mathbb{N}} \subset 2^{\mathbb{N}}$ such that each $A_n \subset A_{n+1} \subset \mathbb{N}$ and $\cup_{n \in \mathbb{N}} A_n = \mathbb{N}$, we have $\lim_{n \in \mathbb{N}} \nu(A_n) = \nu(\mathbb{N})$.

Definition

Given a probability measure μ and $\epsilon \in [0, 1)$, let ν_ϵ be the convex capacity defined by $\nu_\epsilon(A) = (1 - \epsilon)\mu(A)$ for $A \not\subseteq \mathbb{N}$ and $\nu_\epsilon(\mathbb{N}) = 1$. This is called the ϵ -**contamination capacity** with respect to μ .

As introduced by Araujo, Gama, Novinski and Pascoa, utility in (1) gets the weights for every $u(x_j)$ from $C := \text{core}(\nu_\epsilon) \cap \hat{C}$, where $\hat{C}$ is the closed convex hull of

$$\left\{ (\delta_m)_{m \in \mathbb{N}} : \delta_m(j) = \varsigma j + \frac{\beta_i}{j} \text{ for } 1 \leq j \leq m, \delta_m(j) = \varsigma j \text{ elsewhere} \right\}$$

in the weak* topology in ba .

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Economic Interpretation of U^i

Example (The Role of the Infimum)

What is the utility of agent i of receiving $x^i = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{j+1}{j} \right) \right)_{j \in \mathbb{N}}$? Let us start with a finite example where $j \in \{1, \dots, n\}$, $i = 1$ and $u(x_j) = \sqrt{x_j}$. If

- $n = 1$, then $U^1(x^1) = \sqrt{16}(\varsigma_1 + \beta_i)$,
- $n = 2$, then

$$U^1(x^1) = \varsigma_1 \sqrt{16} + \varsigma_2 \sqrt{12} + \beta_i \min_{j \in \{1, 2\}} \left\{ \frac{1}{j} \sum_{j=1}^2 \sqrt{8 \left(\frac{j+1}{j} \right)} \right\},$$

which is equal to

$$U^1(x^1) = \varsigma_1 \sqrt{16} + \varsigma_2 \sqrt{12} + \beta_i \min \{4, 3.732050808\}$$

- $n = 3$, then

$$U^1(x^1) = \varsigma_1 \sqrt{16} + \varsigma_2 \sqrt{12} + \varsigma_3 \sqrt{\frac{32}{3}} + \beta_i \min \{4, 3.732, 3.5766\},$$

- $n \in \mathbb{N}$, then

$$U^1(x^1) = \varsigma_1 \sqrt{16} + \dots + \varsigma_n \sqrt{8 \left(\frac{n+1}{n} \right)} + \beta_i \min_{j \in \{1, \dots, n\}} \left\{ 4, 3.732, \dots, \frac{1}{n} \sum_{j=1}^n \sqrt{8 \left(\frac{n+1}{n} \right)} \right\}.$$

In the example above, we worked with n goods. If we consider countably many of them:

- Instead of the minimum over the j 's, I need the **infimum** over the j 's,
- The infimum might **not** be attained, or attained at **finitely many indices** or attained at a **single (not-cluster) point** (of x^i).

Notice that, since we have monotonous preferences and all goods are preferred, $x_j^i \neq 0 \ \forall j, i \in \mathbb{N}$.

Interpretation: The agent is worried about mean losses, for means computed up to any distant brand. In other words, she is **not sure how introducing less predilected goods into her consumption would affect her mean utility level**. Taking the infimum makes her consider the “*minimum*” mean utility that she would face.

Equilibrium in $\mathcal{E}$

Example

Let $\mathcal{E} = (\ell_\infty^+, U^i, \omega^i)_{i \in \mathbb{N}}$ be defined with

$$U^i(x) = \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^{j-3} \left(\frac{64j+1}{j}\right)^{\frac{-1}{2}} u(x_j) + \beta \inf_j \left(\frac{1}{j} \sum_{k=1}^j u(x_k) \right),$$

$$u(x_j) = \sqrt{x_j},$$

$$\omega_j^i = \begin{cases} (16) \left(\frac{64j+1}{j}\right) & \text{if } i = j, \\ 0 & \text{otherwise,} \end{cases} \quad \forall i \in \mathbb{N}.$$

The equilibrium suggested for the economy is described in the next way: for $\beta = \frac{1}{2}$, with equilibrium allocation $x^i = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{64j+1}{j} \right) \right)_{j \in \mathbb{N}} \forall i \in \mathbb{N}$, the equilibrium prices for $c \in \ell_\infty^+$ are given by

$$\begin{aligned} \pi(c) = & 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{j=1}^{\infty} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j \\ & + \beta_i \left(\frac{1}{2\sqrt{512}} \right) \text{LIM}(\phi(c)), \end{aligned}$$

Important: This illustrates the general property that, in equation (1), if the infimum of consumption of at least one agent is attained at infinity, the AD price functional has a pure charge. This pure charge is, up to a scalar multiple, a generalized limit.

Example (Equilibrium bundles graphically)

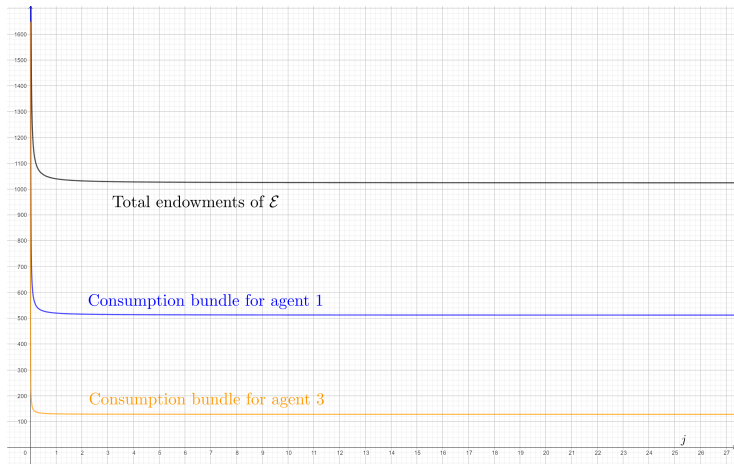


Figure: Total endowments and equilibrium bundles for some agents.

Interpretation of Prices

Usually, in microeconomic theory, the price of a good is equal to the marginal utility of such good. So, it's only natural that the ℓ_1 part of prices, applied to $y \in \ell_\infty^+$, is

$$\mu(y) = \sum_{j=1}^{\infty} \varsigma_j u'(x_j) y_j.$$

However, since agents are unsure on how the consumption of less predilected goods will affect their mean utility, we have

$$\nu(y) = \beta_i u'(\underline{x}) \text{LIM}(\phi(y)).$$

We can interpret it as the marginal utility “*in the worse case scenario of consumption*” scaled by the mean consumption of all of the goods.

Constructing W_k

The steps to construct the withholding coalition are:

- 1 Consider the identity sequence $(i_n)_{n \in \mathbb{N}}$ such that $i_n = n \ \forall n \in \mathbb{N}$,
- 2 Extract from $(i_n)_{n \in \mathbb{N}}$ a subsequence $(e_l)_{l \in \mathbb{N}}$, with $i_{n_g} = e_l$, such that agent n is withholding,
- 3 Since the coalition is made of countably many agents, the set $W_k = \{e_l : l \geq k \in \mathbb{N}\}$ includes all of the agents withholding starting from w_k .

Example

- Tail withholding: $(e_l)_{l \in \mathbb{N}} = m + l - 1$, where m is the starting agent withholding her allocations. Thus, $W_k = \{m + k - 1, m + k, \dots\}$.
 - Evens withholding: $(e_l)_{l \in \mathbb{N}} = 2l$. Thus, $W_k = \{2k, 2(k + 1), \dots\}$.
- Withholders and non-withholders are both countably many.**

Equilibrium in $\mathcal{E}_W$

Following the basis example 8, now we will present two examples of withholding done in different ways:

- 1 Agents in the tail withholding a constant proportion of their endowments,
- 2 Agents indexed by even i 's withholding a constant proportion of their endowments.

Tail Withholding

Example (Tail Withholding)

Let $\mathcal{E}_W = (\ell_\infty^+, U^i, w^i)_{i \in \mathbb{N}}$ be defined with

$$w_j^i = \begin{cases} 16 \left(\frac{64j+1}{j} \right) & \text{if } j = i \text{ and } i < m, \\ 4 \left(\frac{64j+1}{j} \right) & \text{if } j = i \text{ and } i \geq m, \\ 0 & \text{otherwise.} \end{cases}$$

Notice that agents withholding are indexed by $i \in W = \{m, m+1, m+2, \dots\}$, keeping to themselves 75% of their endowments. Let $F = \{1, 2, \dots, m-1\}$ and $S = \{m, m+1, m+2, \dots\}$.

Equilibrium allocations are

$$x^{i^F} = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{64j+1}{j} \right) \right)_{j \in F}, \quad x^{i^S} = \left(\left(\frac{1}{2^{i+3}} \right) 32 \left(\frac{64j+1}{j} \right) \right)_{j \in S} \quad \forall i \in \mathbb{N}.$$

Equilibrium prices for $c \in \ell_\infty^+$ are

$$\begin{aligned} \pi(c) = & 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{j=1}^{m-1} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j \\ & + 4 \left(\frac{1}{\sqrt{2}} \right) \sum_{j=m}^{\infty} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j + \beta_i \left(\frac{1}{16\sqrt{2}} \right) \text{LIM}(\phi(c)). \end{aligned}$$

Notice that these prices are different from those in example 8.

Evens Withholding

Example (Evens Withholding)

Let $\mathcal{E}_W = (\ell_\infty^+, U^i, w^i)_{i \in \mathbb{N}}$ be defined with

$$w_j^i = \begin{cases} 16 \left(\frac{64j+1}{j} \right) & \text{if } j = i \text{ and } i \in O, \\ 1 \left(\frac{64j+1}{j} \right) & \text{if } j = i \text{ and } i \in E, \\ 0 & \text{otherwise,} \end{cases}$$

where $E = \{2n : n \in \mathbb{N}\}$ and $O = \{2n + 1 : n \in \mathbb{N}\}$. Notice that agents withholding are indexed by $i \in E$, keeping to themselves 93.75% of their endowments.

Equilibrium allocations are

$$x^{i^O} = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{64j+1}{j} \right) \right)_{j \in O}, x^{i^E} = \left(\left(\frac{1}{2^{i+3}} \right) 8 \left(\frac{64j+1}{j} \right) \right)_{j \in E}.$$

Equilibrium prices for $c \in \ell_\infty^+$ are

$$\begin{aligned} \pi(c) = & 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^{2n-1} \left(\frac{64(2n-1)+1}{2n-1} \right)^{-1} c_{2n-1} \\ & + 4\sqrt{2} \sum_{n=1}^{\infty} \left(\frac{1}{2} \right)^{2n} \left(\frac{64(2n)+1}{2n} \right)^{-1} c_{2n} + \beta_i \left(\frac{1}{8\sqrt{2}} \right) \text{LIM}(\phi(c)). \end{aligned}$$

Notice that these prices are different from those in example 8.

Back

Persistent Effect on Prices

For the **tail withholding** (finite amount of agents *not withholding*):

- Equilibrium allocation in both $\mathcal{E}$ and $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ are

$$x^i = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{64j+1}{j} \right) \right)_{j \in \mathbb{N}} \quad \forall i \in \mathbb{N},$$

- Equilibrium prices in $\mathcal{E}$ for $c \in \ell_{\infty}^+$ are given by

$$\pi(c) = 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{j=1}^{\infty} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j + \beta_i \left(\frac{1}{2\sqrt{512}} \right) \text{LIM}(\phi(c)).$$

However, notice that the equilibrium prices in $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ for $c \in \ell_{\infty}^+$ are given by

$$\lim_{k \rightarrow \infty} (\pi_{w_k}(c)) = 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{j=1}^{\infty} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j + \beta_i \left(\frac{1}{16\sqrt{2}} \right) \text{LIM}(\phi(c)).$$

For **evens withholding** (infinite amount of agents *not withholding*):

- Equilibrium allocation in both $\mathcal{E}$ and $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ are

$$x^i = \left(\left(\frac{1}{2^i} \right) 16 \left(\frac{64j+1}{j} \right) \right)_{j \in \mathbb{N}} \quad \forall i \in \mathbb{N},$$

- Equilibrium prices in $\mathcal{E}$ for $c \in \ell_{\infty}^+$ are given by

$$\pi(c) = 4 \left(\frac{1}{\sqrt{8}} \right) \sum_{j=1}^{\infty} \left(\frac{1}{2} \right)^j \left(\frac{64j+1}{j} \right)^{-1} c_j + \beta_i \left(\frac{1}{2\sqrt{512}} \right) \text{LIM}(\phi(c)).$$

However, notice that the equilibrium prices in $\lim_{k \rightarrow \infty} \mathcal{E}_{W_k}$ for $c \in \ell_{\infty}^+$ are given by

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Some Useful Definitions

Definition

An **Arrow-Debreu equilibrium (AD)** is defined as a pair (x, π) such that $x \in \ell_{\infty}^+$ is a feasible allocation, π is a linear functional on ℓ_{∞} and, for each i , x^i maximizes U^i in the budget set $\{a \in \ell_{\infty}^+ : \pi(a - w^i) \leq 0\}$.

Definition

Given a dual pair $\langle X, X' \rangle$ and a concave function f on X , we say that $x' \in X'$ is a **supergradient of f at x** if it satisfies the following supergradient inequality

$$f(y) \leq f(x) + x'(y - x), \quad \forall y \in X.$$

We refer to the collection of supergradients as **superdifferential**, denoted by $\partial f(x)$.

A **supporting price** is, up to a scalar multiple, a supergradient of U^i at x^i .

Definition

Define the one-sided directional derivatives $d^+f(x) : X \rightarrow \mathbb{R}$ and $d^-f(x) : X \rightarrow \mathbb{R}$ at x in direction $v \in \ell_\infty$ as

$$d^+f(x)(v) = \lim_{\lambda \downarrow 0} \frac{f(x + \lambda v) - f(x)}{\lambda},$$
$$d^-f(x)(v) = \lim_{\lambda \uparrow 0} \frac{f(x + \lambda v) - f(x)}{\lambda}.$$

Let's have a look now at a characterization for the equilibrium prices. Remember the Yoshida-Hewitt Theorem, from which we can represent any $\pi \in ba(2^{\mathbb{N}})$ as $\pi = \mu + \nu$, where $\mu \in ca(2^{\mathbb{N}})$ and $\nu \in pa(2^{\mathbb{N}})$. Let $\mathbf{l}(n)$ the real sequence whose first n elements are zero and the remaining ones are equal to 1. The next lemma gives information about the supporting prices:

Lemma

If ν is a pure positive charge in $(\mathbb{N}, 2^{\mathbb{N}})$, then

$$\frac{\nu(x)}{\|\nu\|_{ba}} \in [\liminf x, \limsup x], \forall x \in \ell_{\infty}.$$

If for some $\mu \in ca$, $\mu + \nu \in \partial U_i(x)$, then

$$\|\nu\|_{ba} \in \left[\lim_n d^+ U^i(x; \mathbf{l}(n)), \lim_n d^- U^i(x; \mathbf{l}(n)) \right].$$